Algebraic Models of Robot Kinematics, Obtained by Using H – Geometry (Numerical Algebraic Geometry) Methods

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Introduction

A concept of an angle is very important in geometry. The goal of inverse kinematics is to determine the size of angles and how to rotate rods of a manipulator, so that a robot would reach the desired destination.
It is different in h-geometry than in classical geometry-trigonometry:

1. An angle is measured not in radians α (degree), which physically shows the length of a circle arc and changes from zero to transcendental figure

$$\frac{\pi}{2} = 1.5707963268.......$$

and the angle is measured in h-parameter, which physically shows the length of a line segment and changes from 0 to 1.

2. Classical trigonometry widely uses functions of sine and cosine. Their physical meaning is relation between pales of a rectangular triangle (a, b), and the arguments of these functions are the size of an angle measured in radians α. Functions sinα and cosα of classical trigonometry do not have analytical expressions and might be calculated only by using an infinite series. In order to move from degrees to radians, it is necessary to use transcendental figure π.
3. In h-geometry, the size of a rectangular triangle's angle h is determined by using a simple expression

$$h = \frac{a}{a + b} \tag{1}$$

And the parabolic sine-sph, parabolic cosine-cph and parabolic tangent-tph, and their inverse functions have algebraic expressions [1]

$$sph = \frac{h}{\sqrt{h^2 + (1 - h)^2}} = z \qquad h = \frac{z}{z + \sqrt{1 - z^2}} \tag{2}$$

$$cph = \frac{1 - h}{\sqrt{h^2 + (1 - h)^2}} = z \qquad h = 1 - \frac{z^2 - z\cdot\sqrt{1 - z^2}}{2\cdot z^2 - 1} \tag{3}$$

$$tph = \frac{h}{1 - h} = z \qquad h = \frac{z}{1 + z} \tag{4}$$

1. Classical mathematic models of kinematics

Kinematic science of robots or industrial robot manipulators was formed by using classical methods of theoretical mechanics. Theoretical mechanics widely uses classical mathematical methods of vector algebra, where projections of vectors use classical trigonometry methods for the axis of coordinates. Functions sinα and cosα of classical trigonometry (where α is the size of an angle measured in radians) do not have analytical expressions and might be calculated only by using an infinite series.

Therefore, now computers calculates values of these function by using expressions of infinite series.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}........... \qquad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ....$$

$$\operatorname{acos}x = \frac{\pi}{2} - \left(x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 \cdot x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot x^7}{2 \cdot 4 \cdot 6 \cdot 7} + ... \right)$$

$$\operatorname{atan}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} - ....$$

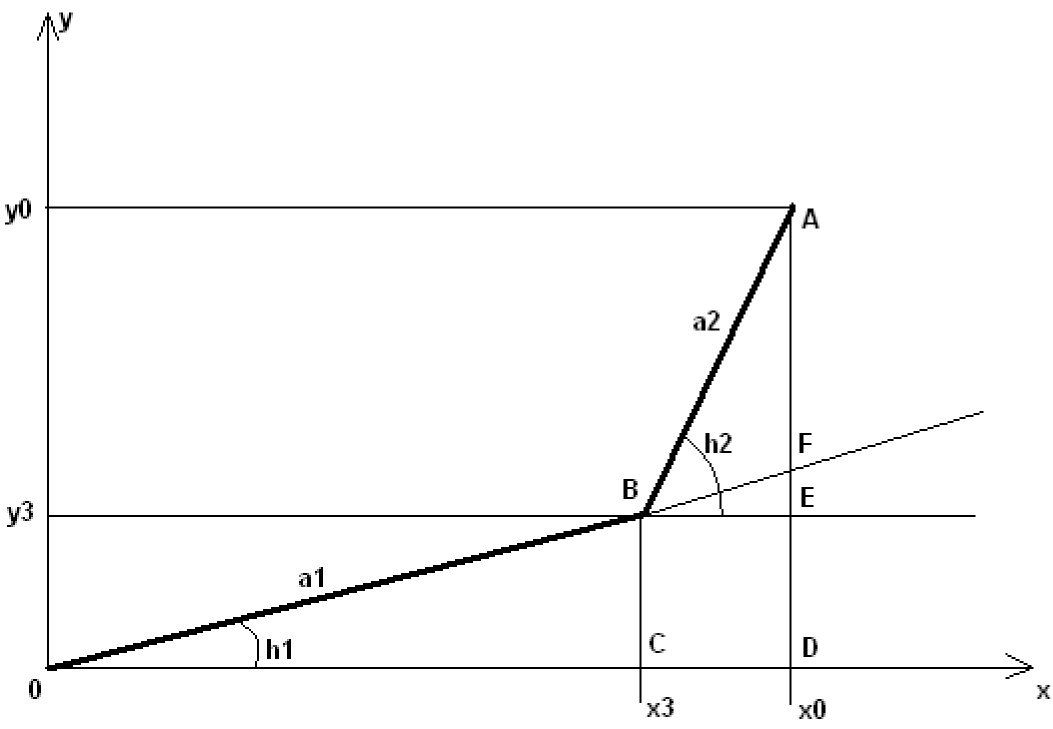
Kinematic models of robots are complex matrices that have elements that are expressions of functions sin and cos.

An angle of a rectangular triangle is the relation between pales of a triangle (a, b)

$$\alpha = \operatorname{atan}\left(\frac{a}{b}\right)$$

While calculating mathematical expressions, obtained from matrices, there are a lot of problems that are emphasized in various article and books. No wonder that in a book [3], and more specifically in the article, of Charles W. Wampler and Andrew J. Sommese "Applying Numerical Algebraic Geometry to Kinematics" they write about algebraic models of kinematics. This particular definition "Numerical Algebraic Geometry" was suggested by the authors in 1996 and discussed at various international conferences [4].

2. The model of inverse kinematics obtained by using the method of h-geometry.



A scheme of simple manipulator is shown in Fig.1. It is required to determine the size h1 of BOC angle and size h2 of ABE angle. They will be calculated by using the expression (1).

$$h1 = \frac{y3}{y3 + x3} \qquad h2 = \frac{(y0 - y3)}{(y0 - y3) + (x0 - x3)} \qquad (5)$$

Coordinates x0 and y0 are given. It is required to determine x3, y3. Known measures are a1 and a2.

In order to determine x3, y3 we will use geometrical (graphical) method. Draw two circles, one of them has point O in the center, and the other has A. Respectively, radius of the circles are a1, a2. Write equations of the first (x1, y1) and second (x2, y2) circle

$$x1^2 + y1^2 = a1^2 \quad (6)$$

$$(x2 - x0)^2 + (y2 - y0)^2 = a2^2 \quad (7)$$

We can write a general equation from both of them, where x and y coincide. The equation will be

$$A \cdot x^2 - B \cdot x + C = 0 \quad (8)$$

where

$$A = x0^2 + y0^2$$

$$B = \left(x0^2 + y0^2 + a1^2 - a2^2\right) \cdot x($$

$$C = \frac{\left(x0^2 + y0^2 + a1^2 - a2^2\right)^2}{4} - y0^2 \cdot a1^2$$

It is understandable that coordinate x3 will be determined from the answer of equation (8)

$$x3 = \frac{B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad (9)$$

Another possible answer of equation (8) will determine a possible coordinate of x4

$$x4 = \frac{B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad (10)$$

Coordinates of y will be determined from the equation of the circle

$$y3 = \sqrt{a1^2 - x3^2} \quad (11)$$

$$y4 = \sqrt{a1^2 - x4^2} \quad (12)$$

When we know values of x3, y3 from (5) we will determine significant values of h1, h2.

3. Example of calculation

Given

$x0 = 1($ $y0 = 6$ $a1 = 8$ $a2 = 5$

From (8), (9), (11), (5) we will determine

$$h1 = 0.15844 \qquad h2 = 0.67885 \quad (13)$$

In order to compare the results by using classical geometry methods, where angles are measured in radians, we will use correlation formula between these systems [5]

$$\alpha = \operatorname{atan}\left(\frac{h}{1 - h}\right) \quad (14)$$

or

$$h = \frac{\tan(\alpha)}{1 + \tan(\alpha)} \quad (15)$$

After using (14) we will determine angles in radians

$$\alpha 1 = 0.186 \qquad \alpha 2 = 1.12893$$

We will use a well-known formula

$$\alpha^o = \frac{180}{\pi} \cdot \alpha$$

We will recalculate their sizes in degrees

$$\alpha 1^o = 10.66275 \qquad \alpha 2^o = 64.68295$$

4. Classical models of inverse kinematics and their modernization

A lot of works for mathematical models of inverse kinematics are written as a system of two trigonometry equations

$$x0 = a1\cdot\cos(\Theta1) + a2\cdot(\cos(\Theta1)\cdot\cos(\Theta2) - \sin(\Theta1)\cdot\sin(\Theta2))$$
$$y0 = a1\cdot\sin(\Theta1) + a2\cdot(\sin(\Theta1)\cdot\cos(\Theta2) + \cos(\Theta1)\cdot\sin(\Theta2)) \qquad (16)$$

They help to determine

$$\Theta2 = \operatorname{acos}\left(\frac{x0^2 + y0^2 - a1^2 - a2^2}{2\cdot a1\cdot a2}\right) \qquad (17)$$

$$\Theta1 = \operatorname{acos}\left[\frac{x0\cdot(a1 + a2\cdot\cos(\Theta2)) + y0\cdot a2\cdot\sin(\Theta2)}{x0^2 + y0^2}\right] \qquad (18)$$

Perform calculation with computer by using expressions (17), (18). We will get

$\Theta2 = 0.9428$ $\qquad \Theta1 = 0.186$

Θ1 corresponds to the angle BOC, which was α1 in our calculations. We notice that the results fully corresponds. The size of the angle ABF corresponds to θ2. In our case this angles is equal to

$$\alpha21 = \alpha2 - \alpha1$$

We notice that α21 fully corresponds to θ2.

The main difference is that while calculating models of h-geometry, calculations are clearly simplified. Since the control of manipulators is perform by microcontrollers, complexity of calculation (time for calculations) is an important parameter.

Lately, classical trigonometry functions and arguments were transformed, which allows to use algebraic expressions. The argument θ was exchanged with the new argument t

$$t = \tan\left(\frac{\Theta}{2}\right)$$

So we can write

$$\cos(\Theta) = \frac{1 - t^2}{1 + t^2} \qquad \sin(\Theta) = \frac{2\cdot t}{1 + t^2}$$

5. Hybrid, classical – h-geometric, model

We know [5] that

$$\sin(\alpha) = sph \qquad \cos(\alpha) = cph$$

If we retain (14), (15), it means that the system of equations (16) might be rewritten with variable h parameters and functions sph and cph

$$x0 = a1\cdot cph1 + a2\cdot(cph1\cdot cph2 - sph1\cdot sph2)$$
$$y0 = a1\cdot sph1 + a2\cdot(sph1\cdot cph2 + cph1\cdot sph2) \qquad (19)$$

where

$$h1 = \frac{\tan(\alpha1)}{1 + \tan(\alpha1)} \qquad h2 = \frac{\tan(\alpha2)}{1 + \tan(\alpha2)}$$

Highlight

$$cph2 = D2 \qquad cph1 = D1$$

where

$$D2 = \frac{x0^2 + y0^2 - a1^2 - a2^2}{2\cdot a1\cdot a2} \qquad (20)$$

$$D1 = \frac{x0\cdot(a1 + a2\cdot cph2) + y0\cdot a2\cdot sph2}{x0^2 + y0^2} \qquad (21)$$

After using reverse function of sph (3), we will determine

$$h2 = 1 - \frac{D2^2 - D2\cdot\sqrt{1 - D2^2}}{2\cdot D2^2 - 1} \qquad (22)$$

$$h1 = 1 - \frac{D1^2 - D1\cdot\sqrt{1 - D1^2}}{2\cdot D1^2 - 1} \qquad (23)$$

As we can see (22), (23) calculations come down to the calculations of algebraic expressions, instead of the calculations of trigonometry functions (17), (18).

References

Donaldas Zanevicius h-Geometry Neo-sines in space mechanics. 2010 Vilnius RDI

Donaldas Zanevičius Mathematics without sin α, cos α (When Angle α is Being Measured in Degrees) and π. *International Journal of Applied Science and Technology* Archve Vol.2 No.6 June 2012 Full Text. http://www.ijastnet.com/update/index.php/current-issue

McCarthy.J.Michael (Ed) 21 st Century Kinematics 2012

The Numerical Solution of Systems of Polynomials Arising in Engineering and Science. Andrew J.Sommese and Charls W.Wamper. World Scientific, 2005

Donaldas Zanevicius h-Geometry Neo-sines in mechanins. 2008 Vilnius. DRI