A Fuzzy Approach to Evaluation of Democracy Index

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Abstract

The present paper is devoted to evaluation of electoral democracy index by using fuzzy logic, non additive measure and Choquet integral. Many works have been done in the area of the evaluation of the democracy, during election. However these methods are not effective as they were based on statistical methods. That is why, the index itself as well as the basic factors of democracy i.e. Stateness, Corruption, Privatization, Multi-Party System, Public Administration, Judicial Independence, Decentralization, regime Type are informations that should be specified by perception, but not by numbers. Information determined by perception can be processed by more adequate methods e.g. by using fuzzy logic theory and Choquet integral. In this paper, a concrete models using fuzzy data is considered.

Keywords: Democracy index, Stateness, Corruption, Privatization, Multi-Party System, Public Administration, Judicial Independence, Decentralization, Regime Type Suffrage, Fuzzy inference, Fuzzy logic, Choquet integral

I. *Introduction*

There are a number of scientific problems in the area of development of the evaluation of democracy index in the election from theoretical and practical view point and improvement of the existing democracy.

The work [1] analyzes factors contributing to democratic consolidation in former Communist States. This paper [1] examines the use of fuzzy subsethood and fuzzy equality operators as tools for assessing factors hypothesized to be associated with successful democratic consolidation in Eastern Europe and the former Soviet Union. Here a data set on factors hypothesized to contribute to the consolidation of democracy is used. To control the greatest number of variables, authors confine their analysis to a single region, post-communist Europe and the former Soviet Union. The hypotheses that they test are: Stateness, Corruption, Privatization, Multi-Party System, Public Administration, Judicial Independence, Decentralization, regime Type. In [2] stochastical electoral models for the Netherlands, Canada and Britain are given, in [3] Elections and legislative politics problems are explained, in [4] a stochastic model of the Russian Duma Election is represented, in [5] the competition for popular support in a stochastic valence model of elections in Turkey is discussed. In work [6] analyzes the model of Politics under proportional representation and Plurality rule and electoral democracy problems. L.Munck and Jay Verkuilen in [7] analyze conceptualizing and measuring Democracy and evaluating alternative indices problems.

The existing work analysis shows that use of statistical methods is not effective to estimate the democracy index. Factors specifying the democracy index are information defined by perception, therefore this information can be operated and processed only using more adequate methods which is based in use fuzzy logic. In [8] modeling electoral democracy index by using fuzzy logic(on the 4 factors) is discussed. The application of an inference algorithm based on fuzzy knowledge processing is strongly required under uncertainty existing in the considered area. For evaluation of electoral democracy index the ESPLAN expert system shell has been used.

In this paper we will discuss the problem of evaluation electoral democracy index using fuzzy logic and Choquet integral. The rest of the paper is organized as follows. In section 2 we give preliminaries. In Section 3 we formulate a statement of the problem. Section 4 is devoted to determination of electoral democracy model using fuzzy rules and Choquet integral. Section 5 is conclusion.

*II. **Preliminaries***

Choquet expected utility model [9]. The Choquet expected utility (CEU) model has the form

$$CEU(f)=\sum_{i=1}^{n}u(f(s_i))w_i$$

Decision weights w_i are non negative. $u(f(s_i))$ - is the utility values.

Possibility measure[10].

Assume that P(X) is a power fuzzy set of the universe X. Then the mapping $\Pi: P(X)\to[0,1]$ with the following properties:

$\Pi(\varnothing)=0,\Pi(X)=1;$

$A\subseteq B\to\Pi(A)\le\Pi(B);$ is the possibility measure. Here I is index set

$\Pi(\underset{i\in I}{\cup}A_i)=\sup_{i\in I}\Pi(A_i).$

*III. **Statement of the Problem***

Here the basic problem is to evaluate electoral democracy index by using fuzzy rules and Choquet integral..

The EDI (Electoral Democracy Index) is a compound index built from eight components each of which is assessed by an expert judge. The eight components are. H1-Stateness, H2- Corruption, H3-Privatization, H4-Multi-Party System, H5-Public Administration, H6-Judicial Independence, H7-Decentralization, H8-regime Type Using the above mentioned eight parameters the electoral democracy model can be expressed as:

Rule 1:IF H1= about 55 and H2=about 20 and H3=about 55 and H4=about 50 and H5=about 20 and H6= about 25 and H7=about 54 and H8=about 55 Then democracy index= about 50 Confidence degree 75

Rule 2:IF H1= about 75 and H2=about 65 and H3=about 70 and H4=about 50 and H5=about 60 and H6= about 55 and H7=about 54 and H8=about 55 Then democracy index= about 68 Confidence degree 50

Rule 15: IF H1= about 35 and H2=about 20 and H3=about 40 and H4=about 20 and H5=about 20 and H6= about 25 and H7=about 26 and H8=about 25 Then democracy index= about 35 Confidence degree 90

Where the value of linguistic variable are trapezoidal fuzzy numbers.

Our aim is to define the level of electoral democracy index using eight democracy index factors represented by fuzzy linguistic terms .

IV**. Determining of Electoral Democracy Index Using Choquet Integral and Fuzzy Rules**

4.1 Determining of Electoral Democracy Index Using Choquet Integral

Applying the formula in Klir and Yuan[11] is obtained:

W=[0.24 0.72 0.24 0.72 0.98 0.24 0.50 0.33]

From the matrix, we conclude that public administration reform (H5) is the factor with the greatest weight in predicting democracy. The factors with the next heaviest weights are corruption (H2) and multiparty system (H4).In paper [11] is determined, that a multiparty system with a weight of 0.72, regime type with a weight of 0.33 , and judicial independence with a weight of 0.24 are the most important factors in democratic consolidation. Corruption with a weight of 0.72, public administration reform with a weight of 0.98 and decentralization with a weight of 0.50 are less important.

Linguistic utility evaluations for the first rule, weights are as shown in the table 1.

For the first rule weights are : W=[0.24 0.24 0.33 0.5 0.72 0.24 0.98 0.72]

The measure g calculated on the base of w is given below:

$\tilde{g}(s_1)$=0.24; $\tilde{g}$ (s1,s2)=0.24; $\tilde{g}$ (s1,s2,s3)=0.33; $\tilde{g}$ (s1,s2,s3,s4)=0.5; $\tilde{g}$ (s1,s2,s3,s4,s5)=0.72; $\tilde{g}$ (s1,s2,s3,s4,s5,s6)= 0.72; $\tilde{g}$ (s1,s2,s3,s4,s5,s6,s7=0.98;

$\tilde{g}$ (s1,s2,s3,s4,s5,s6,s7,s8)=1

Variable	Linguistic utility evalua-tions	utility evaluations (trapezoidal form)	Weights
H1:stateness	About 55	[0,50,60,15]	0.24
H2:Corruption	About 20	[10,10,20,7]	0.72
H3:Privatization	About 55	[10,50,60,11]	0.24
H4:Multi-Party system	About 50	[14,45,60,2]	0.72
H5: Public administration	About 20	[10,10,20,5]	0.98
H6: Judicial Independence	About 25	[20,20,30,1]	0.24
H7: Decentralization	About 54	[20,50,60,2]	0.5
H8:Regime type	About 55	[20,50,60,2]	0.33

The form of a Choquet integral for rule 1 will be:

$$u(\tilde{f}) = (\tilde{u}_1 - \tilde{u}_2)\cdot g(s_1) + (\tilde{u}_2 - \tilde{u}_3)\cdot g(s_1, s_2) + (\tilde{u}_3 - \tilde{u}_4)\cdot g(s_1, s_2, s_3) +$$
$$+(\tilde{u}_4 - \tilde{u}_5)\cdot g(s_1, s_2, s_3, s_4) + (\tilde{u}_5 - \tilde{u}_6)\cdot g(s_1, s_2, s_3, s_4, s_5) +$$
$$+(\tilde{u}_6 - \tilde{u}_7)\cdot g(s_1, s_2, s_3, s_4, s_5, s_6) + (\tilde{u}_7 - \tilde{u}_8)\cdot g(s_1, s_2, s_3, s_4, s_5, s_6, s_7) +$$
$$+(\tilde{u}_8 - \tilde{u}_7) g(\Omega)$$

Using this procedure we have computed the fuzzy values of Choquet integral for the first rule:

U(f)=[-56.32 17.5 98.4 164.12]

Defuz(U(f))=55.6205

Determining of Electoral Democracy Index Using Fuzzy Rules

The mathematical description of knowledge in the knowledge base of decision maker is based on fuzzy interpretation of antecedents and consequents in production rules[12,13].

R^k : *IF* x_1 *is* $\tilde{A}_{k1}$ *and* x_2 *is* $\tilde{A}_{k2}$ *and ... and* x_m *is* $\tilde{A}_{km}$ *THEN*

u_{k1} *is* $\tilde{B}_{k1}$ *and* u_{k2} *is* $\tilde{B}_{k2}$ *and ... and* u_{kl} *is* $\tilde{B}_{kl}$, $k = \overline{1,K}$

Where $x_i, i = \overline{1,m}$ and $u_j, j = \overline{1,l}$ are total input and local output variables ,

$\tilde{A}_{ki}, \tilde{B}_{kj}$ are fuzzy sets, and *k* is the number of rules.

The basic steps of the method are given below:

1. The truth degree of the rule is computed as: $r_{jk} = Poss(\tilde{v}_k / \tilde{a}_{jk})\cdot cf_k$, if the sign is "=" and $r_k = \left(1 - Poss\left(\tilde{v}_k \middle| \tilde{a}_{jk}\right)\right) cf_k$, if the sign is "≠". *Poss* is defined as

$$Poss\left(\tilde{v}\middle|\tilde{a}\right) = \max_u \min(\mu_{\tilde{v}}(u), \mu_{\tilde{a}}(u)) \in [0,1].\ \tau_j = \min(r_{jk})$$

First the objects are evaluated, i.e. every w_i object has appropriate linguistic value defined as (v_i, cf_i). where v_i is linguistic value, $c_{f_k} \in]0,100]$ is confidence degree of the value v_i. v_k - linguistic value of the rule object, a_{jk} - current linguistic value (j is index of the rule, k is index of relation) value(for example , A_ir)

2. For each rule, calculate $R_j = (\min_j r_{jk}) * CF_j / 100$, where CF is the confidence degree of the rule.

The user or the creator of the rule defines the firing level (π) and $R_j \geq \pi$ is checked. If the condition holds true, then the consequent part of rule is calculated.

3. The evaluated w_i objects have S_i value: $w_i, (v_i^1, cf_i^1), ..., ..., (v_i^{S_i}, cf_i^{S_i})$ S_i is the number of the rules in fuzzy inference process

The average value is determined as follows:

$$\bar{v}_i = \frac{\sum_{n=1}^{S_i} v_i^n \cdot cf_i^n}{\sum_{n=1}^{S_i} cf_i^n}$$

IF $x_1 = \tilde{a}_1^j$ AND $x_2 = \tilde{a}_2^j$ AND ... THEN $y_1 = \tilde{b}_1^j$ AND $y_2 = \tilde{b}_2^j$ AND ...

IF ... THEN $Y_1 = AVRG(y_1)$ AND $Y_2 = AVRG(y_2)$ AND ...

This model has a built-in function AVRG which calculates the average value. This function simplifies the organization of compositional inference with possibility measures. As a possibility measure here a confidence degree is used. So, the compositional relation is given as a set of production rules like:

IF $x_1 = \tilde{A}_1^j$ AND $x_2 = \tilde{A}_2^j$ AND ... THEN $y_1 = \tilde{B}_1^j$ AND $y_2 = \tilde{B}_2^j$ AND ,

where *j* is a number of a rule. After all these rules have been executed (with different truth degrees) the next rule (rules) ought to be executed:

IF THEN $Y_1 = AVRG(y_1)$ AND $Y_2 = AVRG(y_2)$ AND ...

Using this model one may construct hypotheses generating and accounting systems. Such system contains the rules:

IF <condition$_j$> THEN $X = \tilde{A}_j$ CONFIDENCE cf_j

Here "$X = \tilde{A}_j$"is a hypothesis that the object *X* takes the value $\tilde{A}_j$.Using some preliminary information, this system generates elements $X = \left(\tilde{A}_j, R_j\right)$, where R_j is a truth degree of *j*-th rule. In order to account the hypothesis (i.e. to estimate the truth degree that *X* takes the value $\tilde{A}_j$) the recurrent Bayes-Shortliffe formula, generalized for the case of fuzzy hypotheses, is used [12]:

$$P_0 = 0$$

$$P_j = P_{j-1} + cf_j Poss(\tilde{A}_0 / \tilde{A})\left(1 - \frac{P_{j-1}}{100}\right)$$

This formula is realized as a built-in function *BS* :

IF END THEN $P = BS(X, \tilde{A}_0)$.

The above mentioned model is realized by using the fuzzy expert system[10] and different test are performed. For example, It is required to determine the output:

IF H1= about 55 and H2=about 20 and H3=about 55 and H4=about 35 and H5=about 20 and H6= about 25 and H7=about 54 and H8=about 55 Then democracy index= ?

Calculation results: about 50(truth of the rule 52%); about 50(truth of the rule 31%); about 35(truth of the rule 10%); about 50(truth of the rule 20%); about 35(truth of the rule 12%); about 50(truth of the rule 30%);

Thus we obtain:

IF Stateness = *about 55 and* Corruption =*about 20 and* Privatization=*about 55 and* Multi-Party System =*about 35 and* Public Administration=*about 20 and* Judicial Independence = *about 25 and* Decentralization=*about 54 and H8*= Regime Type= *about 55 Then democracy index= about 50*

5. Conclusion

In this paper, modeling electoral democracy index by using fuzzy logic is discussed. The basic factors determining the electoral democracy are analyzed. Realization of electoral democracy model using fuzzy rules and Choquet integral has been implemented.

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