A Proposal to Reduce the Biodiesel Exposure to Financial Risks through Derivatives

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1. Introduction

The agricultural sector can face a series of risky situations, mainly involving natural causes. Too much rain, a long draught, or even the action of pests in plantations - all these factors result in yield losses and, under these circumstances, the crops insurance is activated. However, the insurance sector in Brazil is far from protecting the rural producer properly. Cunha [2002] presents the principles that can guideline the creation of a crop insurance that is private, with voluntary participation, and tailor-made to meet the conditions and needs of Brazilian agricultural producers. The creation of an insurance policy with these characteristics can just occur if the Brazilian insurance market is minimally developed. Currently, only five insurance brokers are present in the market and none of them has had more than 6 years of experience in this agricultural module. Thus, it is extremely difficult to sell agricultural insurance due to the producers' lack of information and difficulties of access [Ozaki, 2007, p.85]. Moreover, agriculture can have an excellent harvest instead of a weather disaster – what can also be considered a problem to the producer as this can lead to a possible 'over-production. Under these favorable conditions, production may reach a level that can make the offer higher than the demand. This may result in insufficient revenue to cover the production costs, leading to disastrous consequences for the economy of the country. However, this can be minimized if the producer is insured "against unfavorable price variations", being able to make use of futures contracts and bonds negotiated at BM&F.

The aim of this research work is to present a financial product that can help reduce the exposure to financial risks through derivatives in the agricultural area, focusing on the bio-diesel. This fuel makes use of several oleaginous seeds as raw material and due to the correlation among their price, it is necessary to have a basket consisting of the basic materials in order to better analyze the "insurance" in question.

The option market, a product belonging to the derivatives category, is an instrument that allows minimizing risks in an efficient way. This consists in a type of insurance which ensures the future sale or purchase of a product at a previously agreed price. Therefore, a 'call option' allows the contract holder to buy the product in the future at the previously agreed price whereas the 'put option' allows the sale of the product at the pre-set price.

Still, if the contract can only be exercised at its end, the option is considered European; if it can be put into practice at any moment it is called American. Obviously, this privileged situation promoted by these contracts has a price, which is called the premium, paid when the contract is signed. The quantification of this premium ("insurance premium"), that is, the value of the option, was developed by Black and Scholes [1973] and Merton [1973]. These authors presented a model to assess this insurance which analytical solution for the resulting differential equation is valid only for the European options and in very well adjusted situations. If the option is American and/or exotic, that is, if it involves non-traditional *payoff* (amount of the contract on the due date), numerical methods should be used, being the most common the ones from the *lattice* family (binomial tree), finite differences and Monte Carlo simulation.

As the differential equation involving a basket of raw materials is quite complex, which is the specific case with the production of bio-diesel, it is almost impossible to get a solution without making use of simulation methods. The methodology developed by Longstaff and Schwartz [2001] allows the evaluation of an American option through the Monte Carlo simulation [see Sobol, 1983] in a very flexible and efficient way. This is the main objective of this research work when analyzing the valuation of the American exotic options for some raw material baskets necessary for the production of bio-diesel.

Unlike other sectors, pricing agricultural insurance contracts is not an easy task, mainly as there is no thorough data about the production. Consequently, the difficulty lies in creating an adequate financial product, within the options market, that can guarantee a significant part of the agricultural production.

In all countries where the agricultural insurance has been operated successfully, the State plays a major role in it. There are no reported significant international experiences without the participation of the State; the governments have realized this is the most efficient way to help society face the adversities in agriculture, including the creation of a stabilizing fund to minimize the brokerage risks and, thus, allowing a more intense participation of the agricultural producers in this capital market. In countries such as the United States, Canada, Mexico, Spain, among others, the government subsidize a large amount of the producers' premium and in the United States, besides the premium, the government also subsidize the administrative and operational costs run by the insurance companies.

This more active participation of the State helps establish an efficient anti-cyclic agricultural policy with positive effects on the whole economy. It also allows the State itself to have a more stable tax collection, better inflation control and also to avoid rural exodus. The State is currently present in the bio-energetic programs, such as the Bio-diesel National Program, created on October 30th, 2002 and implemented under the coordination of the Science and Technology Ministry. Bill nr.3368 proposed that 2% of the bio-diesel should be mixed with mineral diesel (and that's where the name B2 derives from) and, in 2006, the requirement was consolidated by the prospect of reaching 10% in the future. Currently, the government purchase bio-diesel through 'Petrobrás', with prices set by auctions.

Bio-diesel is a type of fuel obtained from the chemical reaction of fatty acids (oils and fat) and short chain alcohol, with the presence of a catalizer. This fuel can be obtained from various raw materials from vegetable sources: soybeans, peanuts, cotton, sunflower, castor oil, corn and also from animal sources: bovine, swine, caprine. Its importance stands out in large centers with dense population as it drastically reduces pollution caused by car fumes. It is also important to mention that bio-diesel can be obtained from the frying oil used in cooking. By making use of this oil, the bio-diesel can also preserve the sewage systems as they would be the final destination of this product. We can also include here the social aspects involved in this issue due to the possibility of creating cooperatives for the collection of used frying oil, which can promote a source of income for their members. These are strategic products for the country since the central governments are interested in investing in alcohol and bio-diesel production plants. When the American government visited Brazil on March 31st, a document signed by both countries stated that they were both committed to jointly developing the bio-fuel area ['Foco Economia e Negócios', April 1st, 2007], signaling a fall of barriers for these renewable energy resources. The European governments have also shown interest in investing in industrial plants in Brazil. These energy resources deriving from vegetable sources are essential to reduce our dependence on petrol and in large cities their addition to fossil fuels will certainly cause a lower impact on the environment, with beneficial consequences to the health of the population.

2. Analysis Model

The agricultural sector is exposed to various risky situations, what demands mechanisms to minimize such inconveniences. The derivatives market has an important instrument to minimize risks, which is the options market. Therefore, this contract gives the holder the right (but not the obligation) to buy or sell on a future date at a pre-set price, having only to pay for the premium.

Black-Scholes' model [Black and Scholes,1973 and Merton 1973] to quantify the premium analyzes the stochastic process involving the return of the assets value:

$$dS = \left(\mu - \frac{\sigma^2}{2}\right) S dt + \sigma S dz \tag{1}$$

where S is the assets price, σ and μ represent, respectively, the volatility and trend of these returns and dz, the Wiener's process. By applying the Itô's formula [see Revuz and Yor, 1994, Oksendal, 2000, Musiela and Rutkowiski, 1998] and imposing the condition of neutral risk [Black and Scholes, 1973 and Merton, 1973], we can have the differential equation that gives us the price of the option.

Black-Scholes' model is a reference for us to obtain the option price for the raw materials basket needed to produce bio-diesel. In this work, the basket contains soy, cotton, sunflower and peanut vegetable oils. Although currently these are the most commonly used oils for bio-diesel production, the model to be presented here is highly flexible and can include other raw materials, mainly oils deriving from animal fat, which have very interesting prices, with no mathematical problems.

The preparation of a raw material basket for the production of bio-diesel is important for the analysis of the bio-fuel price due to the fact that its raw material prices are strongly inter-related [see Johnson and Shanno, 1987, p.143-51 and Pagliardi, 2007, cap3]. Therefore, it is vital to present a system that enables the analysis of this basket and offers adequate insurance, thus helping the bio-diesel pricing.

A call or put option can be perfectly adjusted to the objectives of the financial agents who participate in the bio-diesel market. There are two financial products for the basket: an Asian option and a barrier option, both American, as this option can make the cost more accessible to the bond purchaser because the risk is minimized. These two bonds here presented are represented by the American options which differential equations, used for pricing the raw materials baskets (composed of vegetable oils) for the bio-diesel production, can be solved by the Monte Carlo simulation, which is made viable by the algorithm proposed by Longstaff and Schwartz [2001].

2.1 Longstaff and Schwartz' algorithm

In order to understand the intuitive algorithm approach presented by Longstaff and Schwartz [2001], called LS-algorithm, it is necessary to remember that as it is an American option, the holder can, at any time, exercise his or her right. Consequently, in the algorithm, the holder of the American option compares, optimally, the *payoff* (the amount of the payoff on the due date) of the immediate exercise with the expected continuation *payoff* and then exercises his or her right if the immediate payoff is higher. Therefore, the strategy of the optimal exercise is basically determined by the conditional expectation of the continuation *payoff* to keep the option validity. The key issue of the approach is that this conditional expectation can be estimated by "*cross-sectional*" information, through simulation, with the use of the least-squares. Specifically, a regression is made after obtaining the continuation *payoffs* on the functions of the status variables valuation. The adjusted value of this regression offers a direct estimate of the conditional expectation function. By estimating the conditional expectation function for each date of the exercise, it is possible to obtain a complete specification of the optimum strategy to exercise the right along each path. With this specification, the American options can be evaluated in a more adequate way through simulation. This technique is referred to as the LS-algorithm method, which will be described below.
In order to determine the value of an option, the first step of the algorithm is to generate N paths describing a geometric Brownian motion to the asset

$$S(t+\Delta t) = S(t) \cdot \exp\left(\left(r - \delta - 0{,}5\sigma^2\right)\Delta t + \sigma \xi \sqrt{\Delta t}\right) \tag{2}$$

where r is the risk-free rate, δ, the dividend rate, σ the volatility and ξ to represent a random deviation extracted from a normal distribution. The temporal increment is Δt. Each of the N paths are generated with the Brownian geometric method and discretion of the paths in M timesteps for the temporal interval t=0, 1, ..., T. The N paths generate a set of prices that is stored in a NxM matrix set, together with a cash flow matrix C, with the same dimension, initiated with zeroes.

The value of the option in the last timestep t_M can be computed for each of the N paths given by the *payoff* function □($S_{i,M}$, K) and the K exercise of the option (in which $S_{i,M}$ is the value of the asset for the path in the timestep M, that is, $S_M=S(T)$). These values are put in the cash flow matrix $C_{i,M}$. After deducting these values until the initial time is reached the average can be obtained, resulting in the price related to the European option.
The next step of the algorithm is to go through the discreet timesteps, starting in M-1 and decreasing up to the initial point, carrying out the following tasks in each timestep:

For each path i with P($S_{i,M}$, K)>0 in step j (that is, in time t_j), i.e. the path is "*in the money*", the value for the immediate exercise P($S_{i,j}$, K) goes into the e vector.

- The value of continuation is obtained by discounting the interest rate at time j of cash flow matrix C, more specifically $exp(-r(m-j)\Delta t)C_{ij}$ where $m>j$ is the column for line i. The S_{ij} values enter the regression for all the i paths that are "*in the money*", that is S_{ij} is compulsorily considered as positive.
- The regression is carried out by producing a regression function R(S), with the aid of the least squares method. The estimated value of continuation R(S_{ij}) is compared to the value of the immediate exercise. If R(S_{ij})< e_i, that is, if the value of the immediate exercise is higher than the estimated value of continuation, the option is carried out, C_{im} takes value zero and C_{ij} is put into the value of the immediate exercise.

The last step of the algorithm deducts all the non-zero entries in the cash flow matrix, going back to the initial time, and the average of the paths represents the LS-algorithm approximation for all the values of the option.

2.2. Model proposed for the raw materials basket

Derivatives are a family of financial contracts, depending on their respective payoffs, based on the value of some variable that can be the price of equities (stocks), of a stocks portfolio, commodities, interest rates, indexes or even of a generic non-financial asset. Differently from the European options, the Americans are more complex to be analyzed.

In this present work [see Pagliardi, 2007], it is necessary to present the proposed financial product, that is, a model that enables the pricing of the American option for a basket of assets, which is a reference to set the bio-diesel price. As 'Petrobras' acquires the bio-diesel, the government can present a basket of products which will originate the pure oil that, when added to the price estimate of the bio-fuel processing can offer a consensual price to the bio-diesel market.

Here we consider only oils derived from soybeans, cotton, sunflower and peanuts used for the production of bio-fuels. However, the methodology to be used allows adding other raw materials without any serious difficulty. Four assets are considered, based on the following stochastic processes:

$$dS_i = \left(r - \frac{\sigma^2}{2} \right) S_i dt + \sigma_i S_i dz_i \qquad (3)$$

where the index i=1,2,3,4 represents the raw materials, respectively, soy, cotton, sunflower and peanut oils. As it considers the soy oil as the major raw material, it is correlated (ρ_{1j}) with the other raw materials by the relationship:

$$dz_1 dz_j = \rho_{1j} dt \qquad (4)$$

where the index j=2,3,4 represents cotton, sunflower and peanut. Thus, for instance, ρ_{13} means the correlation between soy and sunflower. Yet, with r being the risk-free interest rate and z_i the Wiener process.

The generation of the variables z_1 and z_j , *j*=2,3,4 comes from the normal bivariate distribution with correlation ρ_{1j} that is not obtained by the generation of normal standardized independent variables ε_i , *i*=1,2,3,4, to combine them later so as

$$
\begin{aligned}
z_1 &= \varepsilon_1 \\
z_j &= \rho_{1j}\varepsilon_1 + \sqrt{1-\rho_{1j}^2}\cdot\varepsilon_j \qquad j=2,3,4,
\end{aligned}
\tag{5}
$$

Following the steps of the Black-Scholes model, that is, after applying the 'Itô formula' [see Revuz and Yor, 1994, Øksendal, 2000, Musiela and Rutkowiski, 1998] and imposing a neutral risk condition [Black e Scholes, 1973 e Merton, 1973], the equation that allows determining the value of the option on the products basket can be described as follows:

$$
\begin{aligned}
-\frac{\partial C}{\partial t} &= v_1 S_1 \frac{\partial C}{\partial S_1} + v_2 S_2 \frac{\partial C}{\partial S_2} + v_3 S_3 \frac{\partial C}{\partial S_3} + v_4 S_4 \frac{\partial C}{\partial S_4} \\
&+ \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 C}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 C}{\partial S_2^2} + \frac{1}{2}\sigma_3^2 S_3^2 \frac{\partial^2 C}{\partial S_3^2} + \frac{1}{2}\sigma_4^2 S_4^2 \frac{\partial^2 C}{\partial S_4^2} \\
&+ \rho_{12}\sigma_2 S_1 \sigma_2 S_2 \frac{\partial^2 C}{\partial S_1 \partial S_2} + \rho_{13}\sigma_1 S_1 \sigma_3 S_3 \frac{\partial^2 C}{\partial S_1 \partial S_3} + \rho_{14}\sigma_1 S_1 \sigma_4 S_4 \frac{\partial^2 C}{\partial S_1 \partial S_4} - rC
\end{aligned}
\tag{6}
$$

Applying the method of the finite differences, more specifically the *Alternating Direction Implicit* (ADI) method, requires a mathematical effort to transform the differential equation, simplifying it in a standard equation for heat transfer. In case this procedure is not used, the effort becomes a computing one. Still, as it is an American option, we face a problem of free boundary, thus increasing its complexity. And the problem becomes even more complex with the two products proposed in the research: an Asian option and a barrier option. The Asian option allows minimizing the asset volatility and consequently limits the action of the major groups in the control of asset prices. Regarding the barrier option the idea of the minimum price is present.

In the first case, the barrier can be interesting for the one who sells the oil used in the production of bio-diesel. This seller can be interested in selling this raw material for R$ 1.50 /kg (exercise price). Thus, the suggestion is a *down-and-in put option*, but with a barrier in the amount of R$ 1.30, as it is believed that this minimum price is affordable. This is a put option which will be valid only if the basket price reaches or exceeds the barrier of R$ 1.30. As from that, this producer will sell his/her raw material if the price is below R$ 1.50, which is the option exercise. This is a partial protection which guarantees the producer a minimum price for the raw material in more favorable conditions than the value of a traditional option.

The American-type Asian options are particularly complex because not only do they allows anticipation of the exercise but they are also *path dependent*, as the average A_t depends on the path taken by the prices. In general these problems are difficult to be solved by the technique of finite differences. In these cases we must evaluate the option for the finite differences method, turning the *path-dependent* problem into a Markovian problem. This can be done by introducing the average of the prices as another state variable in the problem. Consequently, the differential equation that rules the Asian option price is added by the terms $\frac{\partial C}{\partial A_i}$, that is,

$$
\begin{aligned}
-\frac{\partial C}{\partial t} &= v_1 S_1 \frac{\partial C}{\partial S_1} + v_2 S_2 \frac{\partial C}{\partial S_2} + v_3 S_3 \frac{\partial C}{\partial S_3} + v_4 S_4 \frac{\partial C}{\partial S_4} \\
&+ \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 C}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 C}{\partial S_2^2} + \frac{1}{2}\sigma_3^2 S_3^2 \frac{\partial^2 C}{\partial S_3^2} + \frac{1}{2}\sigma_4^2 S_4^2 \frac{\partial^2 C}{\partial S_4^2}
\end{aligned}
\tag{7}
$$

$$+\frac{1}{t+t_0}\left[(S_1-A_1)\frac{\partial C}{\partial A_1}+(S_2-A_2)\frac{\partial C}{\partial A_2}+(S_3-A_3)\frac{\partial C}{\partial A_3}+(S_4-A_4)\frac{\partial C}{\partial A_4}\right]$$

$$+\rho_{12}\sigma_2 S_1\sigma_2 S_2\frac{\partial^2 C}{\partial S_1\partial S_2}+\rho_{13}\sigma_1 S_1\sigma_3 S_3\frac{\partial^2 C}{\partial S_1\partial S_3}+\rho_{14}\sigma_1 S_1\sigma_4 S_4\frac{\partial^2 C}{\partial S_1\partial S_4}-rC$$

Subject to an expiration condition type *max*(0, A_T–*S*), in this case with *t* varying in the interval $t_0 \le t \le T$, that represents the interval allowed for the option exercise. The equation (7) represents the differential equation to rule the price of the Asian option for bio-diesel.

As a consequence, the sale of the basket based on its average price (arithmetic average) offers higher price stability as price peaks and slumps are minimized during the term of the contract, with "better behaved volatilities".

The LS-algorithm is a technique that allows solving problems of difficult analysis and complex computing solution, as it happens with the equation (7), being very useful as a flexible method. The option valuation is obtained from:

$$S_i(t+dt)=S_i(t)\cdot\exp\left[\left(r-\frac{1}{2}\sigma_i^2\right)dt+\sigma_i\sqrt{dt}\cdot z_i\right] \qquad (8)$$

where *i*=1,2,3,4 represents the paths of the prices for the considered raw materials. The volatilities, on the other hand, will not present Brownian motion but they will be subject to the variations pre-established according to the observer's view.

The models involving stochastic volatilities, or even stochastic interest rates, require a more careful approach [see Vasicek, 1977, p.177-188, Cox and Ross. 1976, p.145-166, Hull and White,1987, p.281-300, Hull, and White, 1990, p.87-100, Kloeden, Platen and Schurz, 1994, p.264-269]. In this work, the non-constant volatility is considered but with a determining and discreet function to allow the process to be Markovian.

Regarding the prices of the raw materials to determine the volatilities and correlations, these refer to the period from January 2002 to December 2006, available in the '*Agrianual 2005*' (for soybeans) and 'AGPYA 2006' (for the other raw materials). The standard deviations (volatilities) were related to the period from January 2004 to December 2005 (24 data items). However, the correlations were determined based only on year 2005 (12 data items). These values are available in Table 1, together with some relevant statistics concerning the products.

Table.1. Main statistics information on the prices of raw materials for the production of bio-diesel (prices in R$/kg) – January 2002 to December 2005.

Raw materials (oils)	Average Prices	Standard Deviation	Minimum price	Maximum price	Correlation with soybeans
Soybean	1,62	0,396	0,860	2,341	1,000
Cotton	1,43	0,295	0,947	1,907	0,955
Sunflower	1,59	0,247	1,050	2,049	0,913
Peanut	2,63	0,761	1,376	4,207	0,588

As previously pointed out, the volatilities are not considered constant, but they are not stochastic either. These volatilities are represented by deterministic functions, following paths pre-determined by the analyst. In this work, discreet functions are considered to describe the volatilities that are based on the history of the four oils used for bio-diesel production. Monthly volatilities are observed in the period of one year, with the 13 values ("moving volatility") that will be the basis for estimating the annual historical volatility [see Hull, 1992, p.215] to be considered for obtaining a price for the call or put options of the basket for bio-diesel production. The results are shown in Table 2 below.

Table.2 Estimates of the volatility of the oils from the basket to produce bio-diesel (historical volatilities in percentages) – January 2004 to December 2005.

Month/year	Soybean	Cotton	Sunflower	Peanut
Jan 03 - Jan 2004	9,424	9,242	8,894	15,205
Feb 03 - Feb 2004	11,568	9,326	9,349	12,846
Mar 03 – Mar 2004	11,401	8,952	8,658	11,235
Apr 03 - Apr 2004	11,010	7,590	7,232	11,187
May 03 - May 2004	10,790	7,396	7,280	11,541
Jun03 - Jun 2004	11,904	9,918	8,153	10,806
Jul 03 - Jul 2004	11,909	9,929	8,189	11,472
Aug 03 – Aug 2004	11,690	9,934	7,618	10,726
Sept 03 -Sept 2003	11,593	10,196	7,584	10,651
Oct 03 - Oct 2004	10,079	10,036	6,547	9,746
Nov 03 - Nov 2004	10,021	7,959	5,961	7,998
Dec 03 – Dec 2004	9,874	7,123	6,987	6,783
Jan 04 - Jan 2005	9,954	19,383	6,245	6,784
Feb 04 – Feb 2005	7,313	7,878	6,005	6,534
Mar 04 - Mar 2005	11,436	11,598	7,718	6,993
Apr 04 - Apr 2005	11,548	11,604	8,016	7,000
May 04 – May 2005	12,037	11,842	7,750	5,476
Jun04 – Jun 2005	11,620	10,790	7,297	5,505
Jul 04 - Jul 2005	11,544	11,144	7,167	4,202
Aug 04 – Aug 2005	11,459	11,665	7,347	3,993
Sept 04 -Sept 2005	11,477	11,615	7,682	4,316
Oct 04 - Oct 2005	11,031	11,577	7,196	4,295
Nov 04 - Nov 2005	11,019	11,656	7,672	4,213
Dec 04 – Dec 2005	11,0448	11,720	7,407	4,387

Source: Pagliardi, 2007, tab.2, p.104

The discreet value of the raw materials volatility, to be used for estimating prices of the exotic American option, can perfectly match their historical values. However, the analyst can figure out some points with new configuration and change them according to his/her expectations.

It is necessary to compare raw materials prices with the effective production of the oils that are considered components of the bio-diesel, because peanut is the raw material with the highest price whereas soybean accounts for most of the bio-fuel production. These values are presented in Table 3.

Table. 3. Brazilian production of the main oleaginous seeds and their respective vegetable oils for bio-diesel production.

Oleaginous seeds	Production in grains (t)	(%)	Production in oil (1000 t)	(%)
Soybeans	56.865.475	95,0	5.613,2	88,93
Cotton	2.296.186	4,0	558,0	8,84
Sunflower	77.872	0,4	75,8	1,20
Peanut	228.347	0,4	65,0	1,03
Total	59.467.880		6.312,0	

Source:Vegetable production (grains): '*Anuário Estatístico do Brasil'* – 2007.
Vegetable oil production: 'Apud – Bio-diesel Committee from Abiove' – 2003.

The information regarding the production of vegetable oils, that is, the bio-diesel raw materials, was presented by the 'Bio-diesel Committee' from Abiove ─ 2003, at the 1st Brazil-Germany Bio-fuels Forum on November 4th, 2004 in São Paulo and later these data suffered slight adjustments for the vegetable oils basket pricing.
The *payoff* of the presented model is the following:

max[K - (soybeans + cotton + sunflower + peanut), 0]

with soybeans, cotton, sunflower and peanut representing their participation in the production of the oil for bio-diesel, multiplied by their respective prices; K is the price of the exercise. Thus, by considering the values from table 3, we have:

max [K - {0,8893 (soy price) + 0,0884 (cotton price)
+0,0120 (sunflower price) + 0,0103 (peanut price)}, 0].

This *payoff* will be used in both options, the Asian and the barrier options.
The application of the LS-algorithm for obtaining an Asian *put option* and a barrier option, both American, was carried out with a free-risk annual interest rate of 10% and a contract for the period of one year. The volatilities vary monthly, and the reference is the year 2005(see table 2). Four exercises were considered, K= R$ 1.620,00, K= R$ 1.700,00 , K= R$ 1.800,00 and K= R$ 2.630,00 for the two options, and the minimum price for the barrier option was R$ 1.600,00. The current unit prices for the four raw materials refer to the ton, being R$ 1.620,00 for the soybeans, R$ 1.430,00 for cotton, R$ 1.590,00 for sunflower and R$ 2.630,00 for peanut. As the soy oil is the main raw material for the production of bio-diesel, this should be the reference for the correlation among the prices of cotton, sunflower and peanut oils. The correlation between soy and the other raw materials (cotton, sunflower and peanut) is 90%, 95% and 50%, respectively. The LS-algorithm was applied in the two situations with 10 thousand paths in order to make the Monte Carlo simulation feasible, which used the 1st grade polynomial in the regression [see Pagliardi, 2007]. However, the chosen stages were 12 for the Asian option and 20 for the barrier option. Regarding the participation of the raw materials, three scenarios were considered, as shown in Table 4 below.

Table.4. Simulation for bio-diesel raw materials: percentage of the vegetable oils and results for the American Asian put options and barrier options at R$ 1.600,00

Vegetable Oils		Scenario 1	Scenario 2	Scenario 3
Soybeans		88.93%	80%	65%
Cotton		8.84	15	15
Sunflower		1.20	3	15
Peanut		1.03	2	5
Asian	K=1620	R$ 0,37879	R$ 0,25481	R$ 0,018831
	K=1700	R$ 12,7588	R$ 13,1365	R$ 4,03571
	K=1800	R$ 87,5146	R$ 89,9416	R$ 62,5300
	K=2630	R$ 913,147	R$ 915,740	R$ 886,549
Barrier	K=1620	R$ 0,34271	R$ 0,28774	R$ 0,02267
	K=1700	R$ 12,3026	R$ 13,1964	R$ 0,09571
	K=1800	R$ 86,2205	R$ 88,2651	R$ 53,5470
	K=2630	R$ 889,949	R$ 906,179	R$ 310,701

It should be observed that Scenario 1 represents the current situation (see Table 3) with soy contributing to almost 90% of the bio-diesel production. Scenario 2 forecasts an increase in production with a change in the participation of the raw materials, that is, soy still as the major contributor but the other raw materials almost doubling their participation in bio-fuel. It is also possible to notice that the price of the several put options just change significantly when the peanut percentage is altered, since its oil unit price is much higher than the other raw materials, as shown in Scenario 3. However, when the participation of cotton oil increases, the price of the put option also increases due to its lower price when compared to the other components.

If only the soy oil participated in the basket (with 100%), the Asian put option would be at R$ 0,334 and the barrier option at R$ 0,303 for an exercise of R$ 1,620.00. As expected, these amounts are close to the ones presented in Table 4. Still considering this exercise of R$ 1,620.00 per ton of the resulting oil, insurance would currently (Scenario 1) represent 0,023% of this exercise. But, if the basket is sold by R$ 2,630.00 per ton (which is the price of the peanut oil), the insurance value rises considerably, representing 34.72% of the value of the exercise.

Notice that with the participation of peanut oil going from 1% (Scenario 1) to 5% (Scenario 3) the put option price presents high variation if the contracted exercise is at R$ 1,620.00: 20 times for the Asian and 15 times for the barrier option, approximately. Still in the case of an exercise of R$ 2,630.00 (peanut price), the difference between Scenarios 1 and 3 is 1.03 times for the Asian and almost 3 times for the barrier option. We can also notice that when the exercise amount rises, the barrier put option becomes more interesting as it presents a lower premium (insurance) amount.

It is also important to highlight the situation of the peanut oil producer who grows this product mainly. Due to the fact that the price of this oil is 60% higher than the soy oil, this product has difficulties to participate in a large bio-fuels program, as the sale of the product at around R$ 2,630.00 per ton would represent an insurance of around R$ 900,00.

This consists in a warning signal to the use of castor oil for the production of bio-diesel, as it is a noble oil, used by the aeronautics engineering and also for the production of human prosthesis. Its price at the international market is five times higher than the soy price, so there is no reason for using castor oil to be burnt by automotive engines. It is important to mention the possible inclusion of oils from animal fat (bovine, swine and caprine) for the production of bio-diesel as this could be a very interesting option due to its inviting prices. The inclusion of all of them as one single component, or even separately, would be easily implemented into the algorithm proposed in this paper. These elements should be carefully analyzed by the ones in charge of the development of the National Bio-diesel Program, with the effective participation of the State in this process.

3. Conclusion

The work here presented had one specific purpose when analyzing a bio-fuel: the bio-diesel. This can be used by the government as a reference for the acquisition of bio-diesel, through Petrobrás, which pricing, accepted by the government, is under experimental trials. In order to reach this objective, the financial options can be contemplated, analyzing the prices of certain raw material baskets composed of soy, cotton, sunflower and peanut oils, since it is at this phase that we find the major risk faced by the bio-diesel production. If the government collects relevant information regarding the industrial costs involved in the bio-diesel production, they can get the product from the market at a more competitive price as they will have the tools to estimate a "better" price for the raw materials basket used in the production of this bio-fuel.

In order to analyze the financial risk involved in the bio-diesel, we estimate the risk exposure of the raw materials basket which has as a reference the Black-Scholes model, but with flexibility of the major hypothesis when considering the non-constant volatility. For this basket, the correlations existing among the prices were considered, observing that the "insurance" decreases when the prices have a higher correlation among themselves. This means the possibility of complementation or, even, replacement of raw materials.

The results show the importance of the futures market to ensure the revenue and even promote the survival of the company. Should the current participation of the raw materials in the basket be maintained for the production of bio-diesel, the sale of this basket at R$ 1,620.00, which relates to the price of the ton of soy oil, would be insured by 0.023% of the amount of the exercise. Evidently, as the basket put option increases, the put option premium ("insurance") also increases. So, if the basket is negotiated at R$ 2,630.00 per ton, price corresponding to the peanut oil, its premium rises to more than R$ 900.00. This shows the difficulty faced by small producers who grow products which oils have prices much higher than soybeans, to participate in the Bio-diesel Program.

The few simulations presented in Table 4 show that with changes in the oils participation in the basket, the option price ("insurance") can have significant alterations. This is the case of the basket exercise for R$ 2,630.00 per ton, with a premium of R$ 310.00 (Scenario 3). This price is by far lower than the ones obtained at the same conditions in the other scenarios, revealing the importance of an adequate choice of "insurance" for the several situations. There is a lot to be exploited as the chosen option depends on the desired strategy, the producer's profile and the productive structure considered for the raw materials basket.

Lastly, the government should also, apart from contributing to the exemption and simplification of the taxes and financing for the *agribusiness,* subsidize the costs related to brokerage and BM&F, including the premium, as it already happens in industrialized countries, so as to strengthen the Brazilian agricultural sector.

Appendix A

Convergence of the LS-algorithm

Before offering a theoretical justification for the LS-algorithm, it is presented some basic concepts of stochastic calculus applied to finance. Let $(\Omega,\mathcal{F},P)$ be a probability space, where:

Ω : set of results of experiments which is associated with a certain probability;

$\mathcal{F}$: sigma algebra of subsets of W, that is, a set of information available to a certain date;

P: probability measure, i.e. an application that assigns degrees of uncertainty to events.

An option of American type is a non-negative adapted stochastic process $Z=(Z_t)_{t=0,1,...,T}$, whose values Z_n can be understood as the benefit for the owner to exercise the option at any time *n*. The value of the *Z* in the n-th period is given by *Zn* recursive formula:

$$U_T = Z_T$$

$$U_{n-1} = max\{ Z_{n-1}, B_{n-1}\,E[\frac{U_n}{B_n} \mid \mathcal{F}_{n-1}]\}$$

Thus, if $\tilde{U} = (\tilde{U}_n)_{n=0,1,...,T}$ is a supermartingale under P, this is the smallest supermartingale that dominates $\tilde{Z} = (\tilde{Z}_n)_{n=0,1,...,T}$.

Still, let $Z = (Z_n)_{n=0,1,...,T}$ be a non-negative adapted process. The Snell envelope of Z is a stochastic process defined $U = (U_n)_{n=0,1,...,T}$ by

$$U_T = Z_T$$

$$U_n = max\{ Z_n, E[U_{n+1} \mid \mathcal{F}_n]\}$$

Finally, if $\nu_0 = \inf\{n : U_n = Z_n\}$ is a stopping time then $U^{\nu_0} = (U_{\nu_0 \wedge n})_{n=1,2,...,T}$ is a martingale.

These these statements to suggest that in the stopping time ν_0 holds true the following equality:

$$U_0 = E(Z_{\nu_0} \mid \mathcal{F}_0) = \sup_{\tau} E(Z_T \mid \mathcal{F}_0)$$

where τ is in the set of stopping times between 0 and *T*.

We can also say that if $\nu_n = \inf\{j \geq n : U_j = Z_j\}$ is a stopping time then $U^{\nu_n} = (U_{\nu_n \wedge m})_{m=1,2,...,T}$ is a martingale and

$$U_n = E(Z_{\nu_n} \mid \mathcal{F}_n) = \sup_{\tau} E(Z_T \mid \mathcal{F}_n).$$

Definition. A stopping time ν is optimum for an adaptive process $Z = (Z_n)_{n=0,1,...,T}$ if:

$$E(Z_\nu \mid \mathcal{F}_0) = \sup_{\tau} E(Z_\tau \mid \mathcal{F}_0).$$

Clearly ν_0 is an optimum to Z and even more, it is a minimal.

Consider now the Snell envelope, assuming that the decomposition of Doob-Meyer *U* is given by $U = MA$ with *M* being a matingale and *A* a non-decreasing predictable process with $A = 0$.

Proposition 1. The largest optimum stopping time for Z is given by

$$\nu_{\max} = \begin{cases} T & se \quad A_T = 0 \\ \inf\{n : A_{n+1} \neq 0\} & se \quad A_T = 0 \end{cases}.$$

Finally, in the case of Markov chains, the envelope of Snell takes a more simplified form.

Proposition 2. Let us consider $X = (X_n)_{n=0,1,...,\infty}$ a homogeneous Markov chain with state space *E* and transition matrix *P*. Let $\psi : \{0,1,...,T\}xE \rightarrow R_{\geq 0}$ and $Z = (Z_n = \psi(n, X_n))_{n=0,1,...,T}$ a non-negative adapted process. Then, the Snell envelope $U = (U_n)_{n=0,1,...,T}$ has the form $U_n = u(n, X_n)$, with the function $u : \{0,1,...,T\}xE \rightarrow R_{\geq 0}$ defined recursively by

$U(T,x) = \psi(T,x)$
$U(n,x) = max\{\psi(n,x), Pu(n+1,x)$.

LS-ALGORITHM

Let us consider (Ω, $\mathcal{F}$, P) a probability space, $(\mathcal{F}_n)_{n=1,...,T}$ a filtration and $Z = (Z_n)_{n=0,1,...,T}$ a non-negative stochastic process. Here it is assumed that $Z_n \in L^2(\Omega, \mathcal{F}, P)$ for all n = 0,1 ,..., T. As for Snell envelope $U = (U_n)_{n=0,1,...,T}$ for Z is defined recursively by

$U_T = Z_T$

$U_n = \max\{Z_n, E(U_{n+1} \mid \mathcal{F}_n)\}$

and if occurs that

$U_n = \sup_\tau E(Z_T \mid \mathcal{F}_n)$

results in $U_n = E(Z_{\nu_0} \mid \mathcal{F}_n)$ where $\nu_n = \inf\{j \geq n : U_j = Z_j\}$.

Now, you can write recursively the optimal stopping times $(\nu_n)_{n=0,1,...,T}$ by

$\nu_T = T$

$$\nu_n = n \cdot 1_{\{Z_n \geq E[Z_{\nu_{t+1}} \mid F_n]\}} + \nu_{n+1} \cdot 1_{\{Z_n < E[Z_{\nu_{n+1}} \mid F_n]\}} \quad \text{(A.1)}$$

Consider that $X = (X_n)_{n=0,1,...,\infty}$ is a homogeneous Markov chain with state space E and transition matrix P. Assume also that, ψ : $\{0,1,...,T\}$x$E \to R_{\geq 0}$ and $Z = (Z_n = \psi(n, X_n))_{n=0,1,...,T}$ is a non-negative adapted process. From the last proposition occurs that $U_n = u(n, X_n)$ where the function u : $\{0,1,...,T\}$x$E \to R_{\geq 0}$ is defined recursively by:

$u(T,x) = \psi(T,x)$
$u(n,x) = max\{\psi(n,x), Pu(n+1,x)\}$.

As this is a Markov chain it has the strong Markov property, i.e.

$E(Z_{\nu_{n+1}} \mid \mathcal{F}_n) = E(Z_{\nu_{n+1}} \mid X_n)$.

The LS-algorithm introduced by Longstaff and Schwartz [2001] uses the equations (A.1) to calculate ν_0 and so $U_0 = E(Z_{\nu_0} \mid X_0)$. The problem boils down to calculate the conditional expectations $U_n = E(Z_{\nu_{n+1}} \mid X_n)$. If $\{\phi_m^n\}_{m=0,1,...}$ is an orthonormal basis of $L^2(\sigma(X_n), P)$ then

$$U_n = \sum_{m=0}^{\infty} a_m^n \phi_m^n .$$

Defining $U_n^M = \sum_{m=0}^{M-1} a_m^n \phi_m^n$ an approach given by the M first terms, we have the stopping times ν_n^M are obtained by replacing U_n for their approaches to the system of equations (A.1) that define the optimum stopping times,

$\nu_T^M = T$

$$\nu_n^M = n \cdot 1_{\{Z_n \geq U_n^M\}} + \nu_{n+1}^M \cdot 1_{\{Z_n < U_n^M\}} .$$

The only problem that remains is to estimate the coefficients a_m^n with n = 0,1, ..., T and m= 0,1, ..., M. This estimation is performed using $N > M$ simulations with paths w_k defined by

$X(w_k) = \{X_n(w_k) : n = 0,1,...,T\}$,

and assuming that $X_0(w_k) = x$ for all k = 1, ..., T. The estimated coefficients $\hat{a}_{M,N,m}^n$ are determined by some mechanism of estimation from N simulated paths, and these estimated coefficients to construct the approximations $\hat{U}_{N,n}^M$ from U_n^M defined by

$$U_{T,n}^{M} = \sum_{m=0}^{M-1} \hat{a}_{M,T,m}^{n} \phi_{m}^{n}.$$

Thus, we can re-estimate the optimal stopping times for

$$\hat{v}_{T}^{M} = T$$

$$\hat{v}_{n}^{M} = n \cdot 1_{\{Z_n \geq \hat{U}_n^M\}} + \hat{v}_{n+1}^{M} \cdot 1_{\{Z_n < \hat{U}_n^M\}}$$

and a natural approach for U_0 will be $\hat{U}_{N,0}^{M}$.

The mechanism of the LS-estimation algorithm is the least squares, obtaining the coefficients $\hat{a}_{M,N,m}^{n}$ as the solution of the following minimization problem:

$$\min \sum_{k=1}^{N} \left(\sum_{m=0}^{M-1} \hat{a}_{M,T,m}^{n} \phi_{m}^{n}(w_k) - Z_{\hat{v}_{N,n+1}^{M}}(w_k) \right)^2 .$$

This completes the demonstration.

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