

## Adding Flexibility to Flexible Mortgages

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### Mortgages with Graduated Payments

Two-step graduated payment mortgages have been available for some time. Payments start low and increase for the first 5, 7 or 10 years and then level off. We note, however, that if the payments grow over the entire life of the mortgage, the structure of the loan is simpler and the initial payments much lower.

There are two natural ways to structure the payments: 1) each month's payment grows by a fixed annual rate somewhat smaller than the mortgage rate (exponential) or 2) each month's payment is increased by a fixed dollar amount (linear). The formulas for payments, balance due, etc. are included in the Appendix. Here we include examples of each type with the standard mortgage as a base for comparison.

The usual two-step graduated payment mortgages use exponential growth and avoid the large final payments by starting with higher payments and leveling off after 5 to 10 years. In these two respects, the usual two-step graduated payment mortgages are closer to the linear growth method.

**Example.** A \$300,000, 25-year mortgage at 4.5%.

*Base Method:* regular monthly payments of \$1,667.50 (blue in both plots).

**Method 1: Exponential payments increase at a fixed annual rate, Figure 1, left plot.**

1 <sup>st</sup> pmt.	Rate of increase	Last pmt.	Total pmts.
\$1,667.50	0%	\$1,667.50	\$500,250.00
\$1,502.65	1%	\$1,927.63	\$511,907.91
\$1,273.60	2.5%	\$2,372.91	\$530,040.78
\$1,133.52	3.5%	\$2,707.81	\$542,465.93

**Method 2: Linear payments increase by a fixed dollar amount, Figure 1, right plot.**

1 <sup>st</sup> pmt.	Rate of increase	Last pmt.	Total pmts.
\$1,667.50	\$0.00	\$1,667.50	\$500,250.00
\$1,545.50	\$1.00	\$1,844.50	\$508,500.00
\$1,362.50	\$2.50	\$2,110.00	\$520,875.00
\$1,179.50	\$4.00	\$2,375.50	\$533,250.00

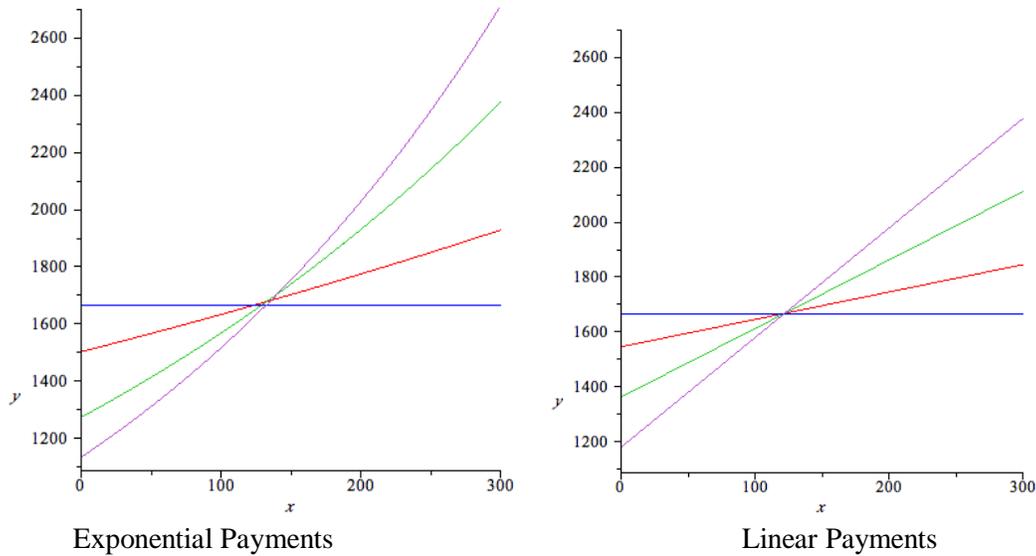


Figure 1

In Figure 2, we plot the balance due on each of the above payment plans.

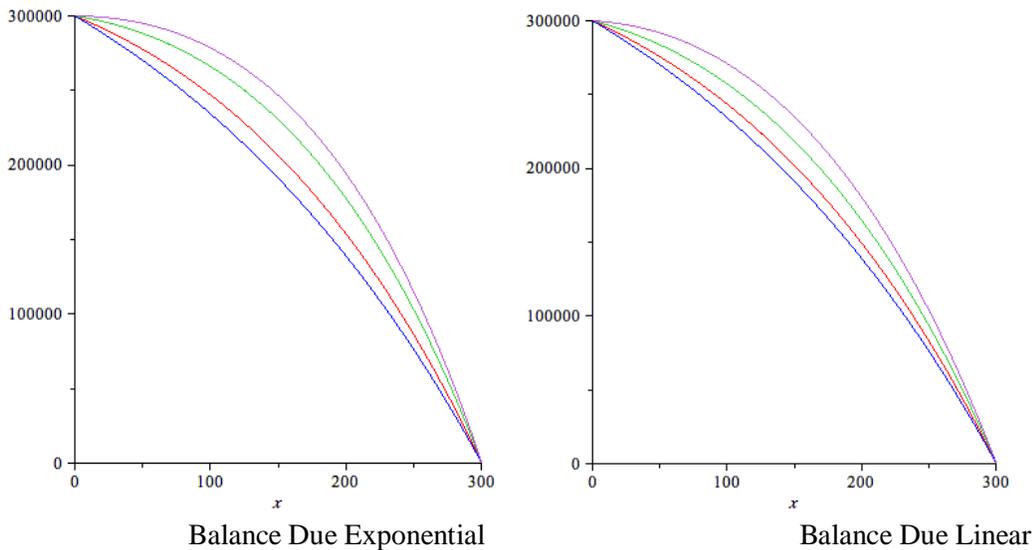


Figure 2

With a minor adjustment, we can easily arrange for the payments to remain constant over each year (which is the case with two-step graduated payment mortgages). The payments will roughly equal the median payment for that year. Each year the payments would increase by the fixed annual rate in the exponential case or 12 times the monthly dollar increase in the linear case.

**Mortgage Certificates of Deposit.** This is a variation on the usual flexible mortgage: set up a savings account at an interest rate tied to the mortgage rate (perhaps 0.5% under the mortgage rate). Withdrawals are automatic mortgage payments – any other withdrawals would carry a penalty. This account would serve several needs. It allows payments on a variety of schedules: automatic weekly or biweekly payments for salaried workers or uneven payments for those with uneven incomes.

**Truly Flexible Mortgages.** A great deal of flexibility becomes possible when a mortgage with graded payments is combined with a mortgage CD. Consider the example given above – a \$300,000, 25-year mortgage at 4.5%. Assume that the mortgagor is employed and can easily manage the \$1,667.50 standard payments but is not too certain about the long-term security of his job. Combining a mortgage CD with a linear graded payment mortgage with \$4 monthly increases, will give the mortgagor a safety net. Paying \$1,667.50 a month into the CD will create a fund that will cover the mortgage payments for a period of time in the case that the mortgagor can't pay.

After just one year, the CD will cover the next 4 mortgage payments and most of the 5th; if the mortgager can make partial payments of \$700 the fund will last a full year. If the mortgager makes the regular \$1,667.50 monthly payments, the CD will continue to grow to about \$42,000 in the 157<sup>th</sup> month. At that time the graded mortgage payments will exceed \$1,667.50 plus the interest on the CD balance and the CD balance will start to decrease. Finally, in the 292<sup>th</sup> month the CD will be depleted and the graded payments will kick in at \$2343.50 and increase by \$4 monthly for the next 8 months. The increased payments in the last year could be avoided by making \$1,680 monthly payments from the start. These are the payments of a standard mortgage at 4.573%. So one could view this arrangement as a standard mortgage with a built in safety net paid for by a 0.073% increase in the interest rate. In this framework, the Australian flexible mortgages have the mortgage CD rate equal to the mortgage rate and the standard mortgage payment structure. Hence the CD balance would remain 0 with no flexibility unless overpayments are made.

**Restructuring.** For various reasons many mortgage holders cannot refinance their mortgage. However, converting the payment schedule of existing mortgage to a graded payment schedule may give the need relief. For example, suppose that the mortgager had taken a \$300,000, 25-year mortgage at 5.75% and 7 years ago. The payments are \$1,887.32 and the balance due at the end of the 7<sup>th</sup> year is \$253,614.14. Borrowing \$253,614.14 at 5.75% for 18 years with exponential graded payments increasing at an annual rate of 3.5% reduce the first new payment to \$1,432.37.

### Appendix: The Basic Formulas.

#### Notation.

- $N$ , the length of the mortgage in months;
- $i$ , the monthly mortgage rate;
- $j$ , the monthly payment rate of increase (exponential);
- $d$ , the monthly payment dollar increase (linear);
- $L$ , the amount of the mortgage;
- $B_n$ , the balance due at the end of the  $n^{\text{th}}$  month;
- $P_n$ , the  $n^{\text{th}}$  payment.

#### Exponential Growth Formulas.

$$L = \left[ \frac{(1+i)^N - (1+j)^N}{(i-j)(1+i)^N} \right] P_1;$$

$$P_n = (1+j)^{n-1} P_1;$$

$$B_n = (1+i)^n L - \left[ \frac{(1+i)^n - (1+j)^n}{i-j} \right] P_1;$$

$$\text{Total payments} = \left[ \frac{(1+j)^N - 1}{j} \right] P_1.$$

#### Linear growth Formulas.

$$L = \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] P_1 + \left[ \frac{(1+i)^N - (1+Ni)}{i^2(1+i)^N} \right] d;$$

$$P_n = P_1 + (n-1)d;$$

$$B_n = (1+i)^n L - \left[ \frac{(1+i)^n - 1}{i} \right] P_1 - \left[ \frac{(1+i)^n - (1+ni)}{i^2} \right] d;$$

$$\text{Total payments} = NP_1 + \frac{N(N-1)}{2} d.$$